Maths activity: Graphs and tangent practice.

Skills: Maths, revision technique.

Read through the attached pages using your note-taking skills

Complete practice questions

Produce a mindmap/flashcards answering the following questions

- ✓ What do line graphs show?
- ✓ How do you draw a line graph or scattergram?
- ✓ How do you use line or scatter grams to spot trends?
- ✓ What do scattergrams show?
- ✓ How do you calculate rate using a straight tangent?
- ✓ How do you calculate rate using a curved tangent?

Have you CONDENSED your notes or just copied out the notes?

Which is more effective?

Make a note of anything you are unsure of and bring this to a tutorial.

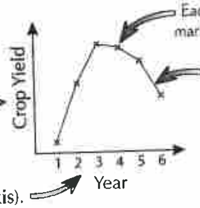
Line Graphs

*Line graphs use **points** connected by **lines** to show how something is changing. They can show as many sets of data as you like, so you can use them to **compare** results. Amazing, I know.*

Line Graphs Are Great For Showing How Something Changes Over Time

Line graphs show how one variable **changes** as another variable (often time) changes. Here's what one might look like:

The thing that was measured (dependent variable) goes up the side (on the y-axis).



The thing that was changed (**independent variable**) goes along the bottom (on the x-axis).

On this graph, the line goes **up** for the first two years. It then goes **down** for the three years after that. So the **crop yield increased** for two years then **decreased** for the next three years.

Plot the Points Then Join Them Up

If you need to draw a line graph of your results, here's what you do:

1) Decide which **variable** should go on each **axis**.

Time should always go along the x-axis.

Glucose concentration is the variable that was measured, so it should go up the y-axis.

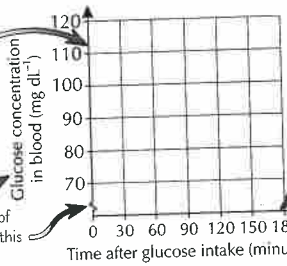
Time after glucose intake / minutes	0	30	60	90	120	150	180
Glucose concentration in blood / $mg\,dL^{-1}$	76	118	100	95	82	74	71

2) Draw your axes so that they've got a sensible **scale**.

The y-axis needs to include numbers from 71 to 118. Here, it works well to have each big square worth 10 $mg\,dL^{-1}$.

Don't forget to label each axis and add the units (if there are any).

If there's nothing happening on the bottom part of the y-axis, you can skip it with a little squiggle like this and start the numbering at a higher value.

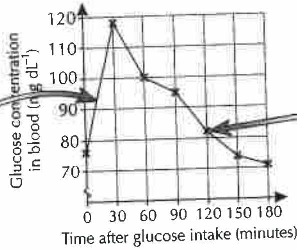


The glucose concentration was measured in 30 minute intervals. So it makes sense to spread the intervals a big square apart on the x-axis.

When it comes to plotting graphs, it's much easier if you've made each square a nice value.

3) **Plot** the points with neat little crosses. Then draw **straight lines** to connect each point to the one next to it.

Make sure you use a sharp pencil to plot the points accurately, and use a ruler to draw the lines.



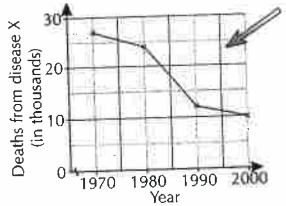
On the y-axis, the scale means that every small square is worth 2 $mg\,dL^{-1}$.

So to plot this point, first find 120 on the x-axis. Then count up one small square from 80 on the y-axis to get 82. Make a neat cross where the two points meet on the graph.

Line Graphs

Line Graphs Are Good For Spotting Trends in Data

To spot a trend in your data, just look for any patterns in the direction of the line:



The line on this graph is constantly going **down**, but at **different rates**. So you can state that:

"The number of deaths from disease X decreased between 1970 and 2000 from 27 000 deaths to 10 000 deaths."

"The number of deaths decreased the fastest between 1980 and 1990, from 24 000 to 12 000 deaths."

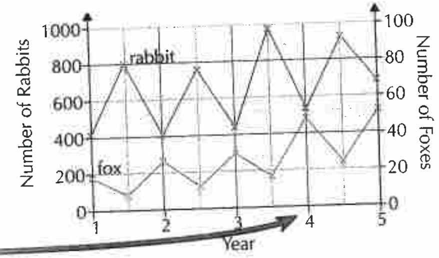
It's a good idea to include relevant values in your description.

Wesley's ability to spot trends was always slightly off.

Worked Example

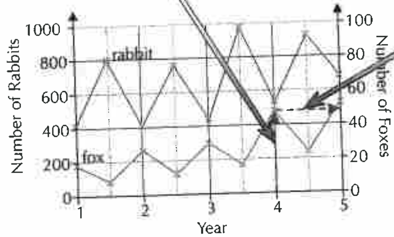
The graph on the right shows the numbers of rabbits and foxes in an area of parkland over a period of five years.

How many foxes were in the parkland at the beginning of year 4?



1. ***Find the axes that you need to look at.***
   The variable that you're given a value for is the **year**, so you want to look at the x-axis first.

2. ***Draw a line up the graph from the x-axis value.***
   The number of foxes is shown by the green line. So draw a straight line up to the green data point.



3. ***Draw a line across to the other axis to find the answer.***
   You want to know the number of **foxes**, so you need to read the value from the y-axis on the right-hand side of the graph.

   Each small square on this axis is worth 4.

   Pay attention to the scale on graphs and charts — if there's a scale on both sides, they won't always go up in the same steps.

   The number of foxes at the beginning of year 4 was **48**.

Practice Question

Q1 A study was carried out into the mortality rate from heart disease in one population between 1980 and 2000. The results are shown in the table.

a) Draw a line graph of the results.

b) Describe the trend shown by the graph.

c) Use your graph to estimate the mortality rate for heart disease in 1989.

Use the line between data points to estimate a value from a line graph.

Year	Mortality Rate / Deaths per 10 000
1980	189
1985	176
1990	171
1995	152
2000	140

I don't mind tables, but graphs are where I draw the line...

Line graphs are pretty easy to handle — as long as you pay a bit of attention to what's on each axis, you'll have no problems understanding what's going on. If the graph has more than one y-axis, just make sure you read off the right one.

Scattergrams and Correlation

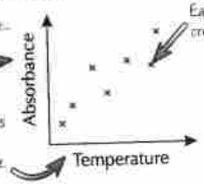
*Scattergrams show the **relationship** between two variables, like the number of friends you have and the number of chocolate fountains in your house. If there's a relationship, you can say that there's a **correlation**.*

Correlation is the Measurement of a Relationship

In a scattergram, one variable is plotted against the other, allowing you to easily spot a **correlation**. Both variables have to be **numbers**. Here's what a scattergram might look like:

Scattergrams can also be called scatter graphs or scatter diagrams.

Scattergrams can show two independent variables.



If you look at this scattergram, you can see a relationship — both variables go up together. As the temperature increases, absorbance also increases.

Plot the Points But Don't Join Them Up

If you need to **draw** a scattergram from a table full of data, here's what you do:

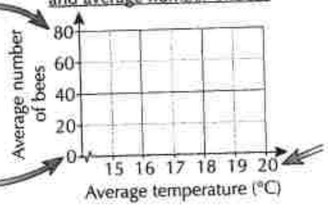
1) Look at the highest and lowest values for each variable to work out a **scale** for each axis.

Average temperature / °C	Average number of bees
17.0	48
19.2	24
15.0	72
18.4	64
17.6	36
16.2	64

The values for this variable range from 15.0 °C to 19.2 °C. A scale from 15 to 20 will do nicely.

The values for this variable range from 24 to 72. So a scale from 0 to 80 will be nice and neat.

2) Draw your axes and **label** them.



Your graph should have a title — just write a description of what the graph is showing.

On the y-axis, the scale means that every small square is worth 4 bees.

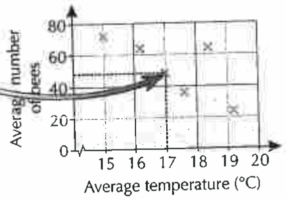
The lowest temperature you need to plot is 15.0 °C, so you can skip out 1-14 using a little squiggle.

On the x-axis, the scale means that every five small squares is worth 1.0 °C.

3) Plot each point with a **sharp pencil**. Draw each point as a neat little cross.

Average Temperature / °C	Average Number of Bees
17.0	48
19.2	24
15.0	72

This pair of values gives you one data point.

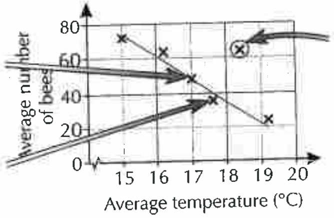


Scattergrams and Correlation

4) On a scattergram, the points are never joined together. However, you may have to draw a **line of best fit**.

Draw the line of best fit as close to as many points as possible.

It doesn't need to go through all the points.



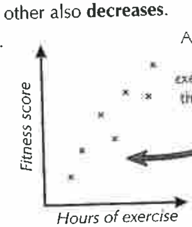
Anomalous results don't fit in with the rest of the data — just ignore them when you're drawing a line of best fit.

Margaret's method of getting to the shops was fairly anomalous.

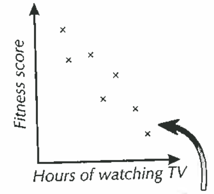
There Are Different Types of Correlation

If you're interested in the **relationship** between two variables, look at the **correlation**. There are a few different types:

1) **Positive correlation**
This means that as one variable **increases**, the other also **increases**. Likewise, if one **decreases**, the other also **decreases**.
E.g.

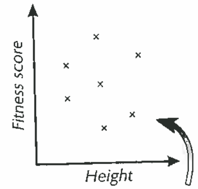


2) **Negative correlation**
This means that as one variable **increases**, the other **decreases**.
E.g.



As the number of hours of watching TV increases, the fitness score decreases.

3) **No correlation**
This means that the variables aren't linked — there is no pattern in the data.
E.g.

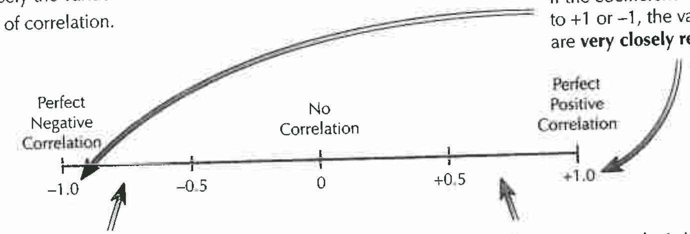


There's no pattern at all to the plotted points here.

The **closer** the points are to the line of best fit, the **stronger** the correlation. A **correlation coefficient** can also be calculated (see p. 69) and this gives you a numerical value for how **strong** the correlation is. It'll always be a number between –1 and +1, and shows:

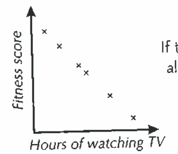
- How closely the variables are **linked**.
- The **type** of correlation.

If the coefficient is close to +1 or –1, the variables are **very closely related**.



If the coefficient is between 0 and –1, it means that the variables are **negatively correlated**.

If the coefficient is between 0 and +1, it means that the variables are **positively correlated**.



If the coefficient is exactly –1.0, all the points lie on a straight line — the correlation is perfectly negative.

If the coefficient is exactly +1.0, all the points lie on a straight line — the correlation is perfectly positive.

Scattergrams and Correlation

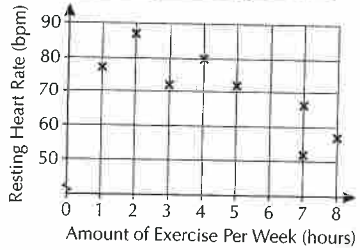
Worked Example

Zoë is investigating the relationship between the time spent exercising per week and resting heart rate in her classmates. Her results are shown in the scattergram on the right.

a) **One of her classmates had a resting heart rate of 80 bpm. How many hours per week did they spend exercising?**

b) **Use the scattergram to comment on the relationship between the time spent exercising and the resting heart rate.**

Relationship between time spent exercising and resting heart rate

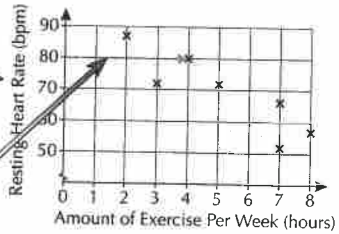


1 ***To answer part a), find the axis that you need to look at.***

The variable mentioned in the question is the resting heart rate, so you want to look at the *y*-axis.

2 ***Draw a line across the graph from the value in the question.***

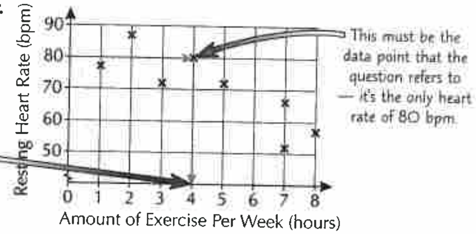
Draw the line across until you meet the data point.



3 ***Draw a line down to the other axis to find the answer.***

Sometimes you might have to read up and across instead.

The classmate spent 4 hours per week exercising.



4 ***To answer part b), look at the correlation.***

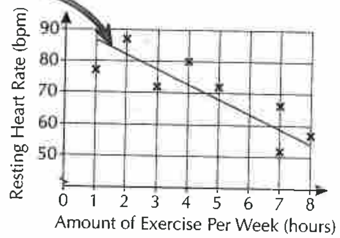
It's a good idea to draw a line of best fit on the graph to help you. The graph shows a fairly strong **negative correlation**, so you can say that:

> **As time spent exercising per week increases, the resting heart rate decreases.**

It's really important to note that you **can't** say something like:

> Increasing the time spent exercising causes a decrease in the resting heart rate. ✘

Although there is a negative correlation, there is no way to tell from the graph if one variable actually **causes** the other.



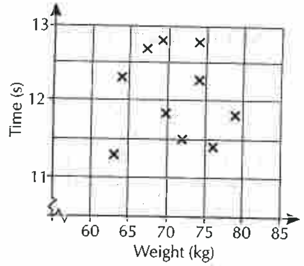
Scattergrams and Correlation

Practice Questions

Q1 A group of athletes were weighed and the time it took each of them to run 100 metres was recorded. The data is shown in the scattergram on the right.

a) How many athletes are represented by the scattergram?

b) What does the scattergram tell you about the relationship between the athletes' weights and the time it took them to run 100 metres? Explain your answer.

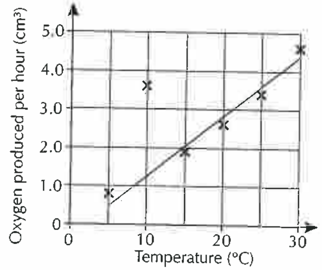


Q2 Charlotte was studying the effect of temperature on the rate of photosynthesis. She decided to use the amount of oxygen produced by a plant in one hour as a measurement of the rate of photosynthesis. The scattergram on the right shows her results.

a) How much oxygen was collected in the experiment conducted at 20 °C?

b) What type of correlation is shown by the results?

c) Charlotte identified one anomalous result on her graph. At what temperature was that result recorded?



Q3 A team of biologists investigated the use of lichen diversity as an indicator of air quality. They calculated an index of diversity for lichens found on gravestones at different distances from a city centre. They predicted that the closer the gravestones were to the city, the lower the diversity of the lichens growing on them would be. Their results are shown in the table on the right.

Distance from city centre / km	Index of diversity
0	0.17
2	0.20
9	0.15
16	0.41
20	0.40
24	0.50
27	0.59

a) Draw a fully labelled scattergram of the results in the table. Include a line of best fit.

b) Using your line of best fit, predict the index of diversity of lichens found on gravestones 10 km from the city centre.

c) The correlation coefficient for the data is 0.86. What does this tell you about the relationship between the diversity of lichens and the distance from the city centre?

All my relationships seem to go negative pretty fast...

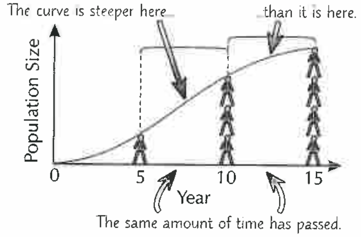
An important thing to remember about correlation is that just because there's a relationship between two variables, it doesn't mean that one has a direct effect on the other — it could just be a coincidence. Correlation doesn't mean cause.

Rates and Gradients

*Rates are pretty important in Biology and come up a lot — you've probably heard of breathing rate, growth rate or rate of reaction. Rate is simply a **measure** of how much something is changing over **time**.*

A Bigger Change Over Time Means a Faster Rate

If you plot your results on a graph with time on the x-axis, the **steepness** of the line (or curve) shows how fast the rate is. Just look at this:



The population increases **more** between years 5 and 10 than between years 10 and 15. There is more growth over the same amount of time. So the **rate** of population growth is higher between years 5 and 10.

Do This Calculation to Work Out the Rate

You might need to calculate a rate from a **table**. If so, you just need to use this equation:

$$\text{rate} = \frac{\text{change in dependent variable}}{\text{time taken for change}}$$

The dependent variable is the variable that's measured — e.g. height.

The change is a positive number if it's an increase, or a negative number if it's a decrease (which will give a negative value for the rate).

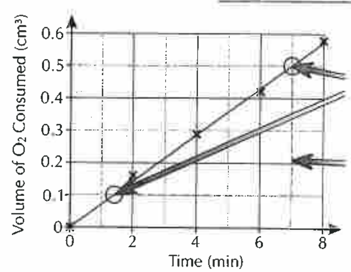
You might have to work out the difference between two values for each of these bits.

To calculate the rate from a **line graph** or a **line of best fit**, you need to work out the **gradient** of the line. The gradient is a value that tells you how **steep** the line is. You find it like this:

$$\text{rate} = \text{gradient} = \frac{\text{change in } y}{\text{change in } x}$$

This is the change in the y-axis (the dependent variable)

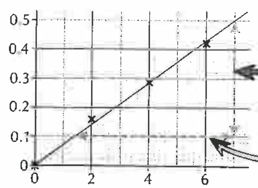
This is the change in the x-axis, which is the time taken for the change.



Pick two points on the line that are easy to read.

Then draw a vertical line down from one point and a horizontal line across from the other to make a **triangle**.

Try to make your triangle at least half the size of the graph.



The vertical side of the triangle is the change in *y*.
So here, it's 0.5 – 0.1 = **0.4 cm³**.

The horizontal side of the triangle is the change in *x*.
So here, it's 7.0 – 1.4 = **5.6 minutes**.

The growth rate of Bruno's ears was beginning to become a concern.

Rates and Gradients

Plug the Values into the Formula

You've now got the two values you need to plug into the gradient formula.

$$\text{rate} = \text{gradient} = \frac{0.4}{5.6} = \mathbf{0.07}$$

Rates always need to have **units**, and these are always:

$$\text{units} = \frac{\text{units of the dependent variable}}{\text{units of time}}$$

Here they're $cm^3\ min^{-1}$, so the rate is **0.07 $cm^3\ min^{-1}$**.

This means that the volume of O_2 consumed is increasing at a rate of 0.07 cm³ per minute.

You might also see these units written as cm³/min — they both mean cm³ per minute.

Worked Example 1

The number of people living in a village in Cumbria was recorded every ten years as part of a 40 year study.

a) **What was the population growth rate between 1960 and 1970?**

b) **Was this higher or lower than the average population growth rate over the course of the study?**

Population	1114	1237	1882	2795	4909
Year	1950	1960	1970	1980	1990

1. ***To answer part a), work out the change in the population (the change in y).***

In 1960, the population was 1237.
In 1970, the population was 1882.

1882 – 1237 = **645 people**

If the population was decreasing, the change would be a negative number.

Population	1114	1237	1882	2795	4909
Year	1950	1960	1970	1980	1990

2. ***Work out the change in time (the change in x).***

The difference in the two dates is **10 years**.

3. ***Divide the change in the population by the time taken for the change to get the rate.***

645 people ÷ 10 years = 65 people $year^{-1}$

Don't forget to include the units for the rate. You can include the units throughout your calculation to help you work it out.

The exact answer is 64.5, but you can't have 0.5 of a person so round the answer to the nearest whole number.

4. ***To answer part b), repeat steps 1-3 to calculate the population growth rate across the full 40 years.***

3795 people ÷ 40 years = **95 people $year^{-1}$**

The change in the population (4909 – 1114 = 3795).

The number of years taken for the change.

Now you can compare the two growth rates.

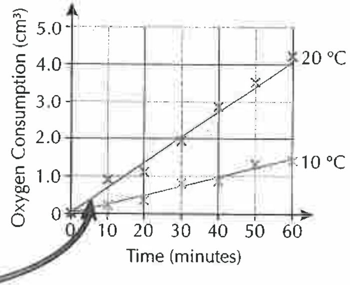
95 is **bigger** than 65, so the population growth rate between 1960 and 1970 was lower than the population growth rate across the whole study.

Rates and Gradients

Worked Example 2

Janet was investigating the effect of temperature on the respiration rate of germinating seeds. She used oxygen consumption as a measure of respiration. Her results are shown in the graph on the right.

Calculate the rate of oxygen consumption for seeds germinating at 20 °C.



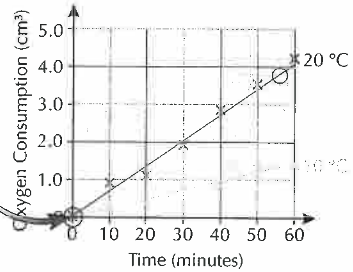
1 ***Find the line you need to look at.***

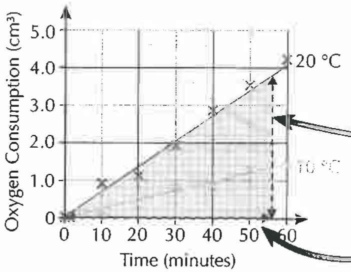
The question asks about seeds germinating at 20 °C, so you need to look at the orange line.

2 ***Find two points on the line that are easy to read.***

If the line goes through O on both axes (the origin), you can use that as one point — it will make the next step a bit easier.

You want to use points that have a clear x and y value, so use points where the line crosses both grid lines at once if you can.





3 ***Make a triangle using the two points. Then work out the length of its side and base.***

The scale on the y-axis means that every small square is worth 0.2 cm³.

So the **height** of the triangle is: 3.8 – 0 = **3.8 cm³**

The scale on the x-axis means that every small square is worth 2 minutes.

So the **base** of the triangle is: 56 – 0 = **56 minutes**

O makes a nice easy calculation.

4 ***Use the equation to find the rate of oxygen consumption.***

The rate is equal to the **gradient** of the line. So put the values from step 3 into this equation:

$$\text{gradient} = \frac{\text{change in } y}{\text{change in } x}$$

This is the height of the triangle.

This is the base of the triangle.

rate = gradient = 3.8 cm³ ÷ 56 minutes = **0.068 cm³ min⁻¹ (to 2 s.f.)**

Round your answer to a sensible value.

Don't forget to include the units — it'll always be one unit over the other for rate.

Rates and Gradients

Practice Questions

Q1 Josie was investigating the rate at which amylase breaks down starch. She used the concentration of maltose as a measure of how the reaction was progressing. Her results are shown in the table below.

a) The rate of reaction can be found by calculating the rate at which maltose is produced by the reaction.
Write down the units for the rate of reaction.

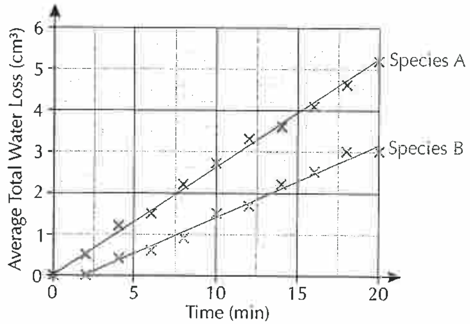
b) Calculate the rate of reaction for the first 20 minutes of the experiment.

Time / minutes	0	5	10	15	20	25
Maltose Concentration / g µl⁻¹	0.0	5.2	10.7	15.6	19.1	25.2

Q2 An investigation into transpiration rate was carried out using two different plant species. The average total water loss per 100 g for each species was recorded every two minutes. The results are shown in the graph on the right.

a) Which plant species had a faster transpiration rate? Explain your answer.

b) Use the graph to calculate the transpiration rate for Species B.



Q3 Kyle was investigating the effect of auxin concentration on plant growth. He removed the tips from three different shoots and coated each with a different concentration of auxin. The table on the right shows a summary of his results.

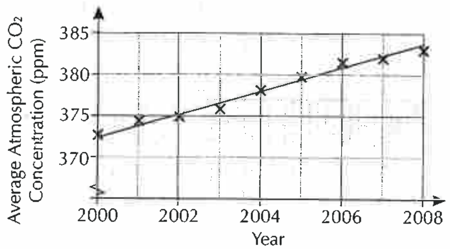
a) What was the average growth rate for Shoot 1? Give your answer in mm hour⁻¹.

You might need to work out a rate in different units to the units given in the data. It's okay though — just convert each value before you pop it into the equation. There's more about converting units on pages 8-9.

b) Which shoot had the highest growth rate?

	Height / cm		
Days of growth	Shoot 1	Shoot 2	Shoot 3
0	1.2	3.2	3.5
5	1.8	4.9	4.1
10	2.6	7.0	4.9
15	4.2	9.2	6.0
20	5.9	11.6	6.9
25	8.2	13.9	7.1
30	9.9	16.3	8.2

Q4 The graph on the right shows the average atmospheric CO_2 concentration in a forest, recorded over a period of 8 years.
At what rate did the CO_2 concentration increase?



My rate of learning about graphs is at an all-time high...

Being able to calculate a rate can come in really handy, so keep practising until you can do it blindfolded, at night, upside down... you get the picture. Just make sure to always include the units, or your answer won't make much sense.

Rates and Tangents

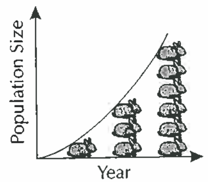
*A tangent is a **straight line** that touches a **point** on a curve. It doesn't sound very exciting, but drawing a tangent happens to be a nifty way of working out a **rate** from a curved line on a graph.*

If you need a reminder about rate, turn back to page 50.

The Rate Can Change Over Time

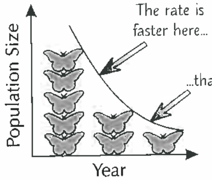
When a variable is changing over time, the **amount** of change over time (the rate) doesn't always stay the same. On a graph, this can give you a lovely **curved line**.

The rate can **increase**...

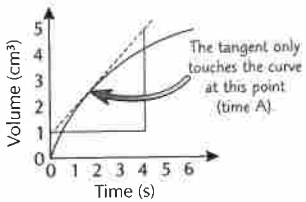


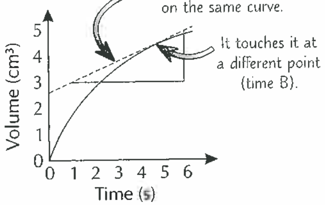
Remember, a steeper curve means a faster rate

...or **decrease**.



You need a **straight line** to be able to calculate a rate from a graph (see p. 50). So to find the rate at a particular time, you can use a **tangent** — like this:





Remember, gradient = rate = $\frac{\text{change in } y}{\text{change in } x}$

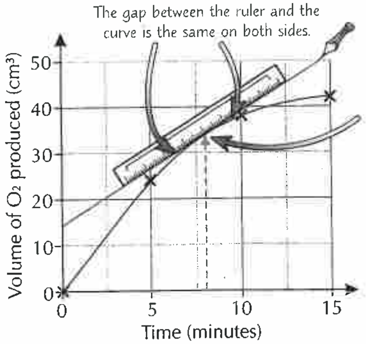
The rate at time A is 4 cm³ ÷ 4 s = 1 cm³s⁻¹.
The rate at time B is 2 cm³ ÷ 5 s = 0.4 cm³s⁻¹.

So the rate of reaction is faster at time A than at time B — a higher volume is being produced per second.

You Need a Ruler to Draw a Tangent

To draw a tangent to a curve on a graph, you just need to follow these steps:

1) Find the point on the graph where you want to know the rate.
2) Put a ruler on the graph so that it's **just touching** that point.
3) Angle the ruler so that there is **equal space** between the ruler and the curve on either side of the point.
4) Draw a line along the ruler using a **sharp pencil**.

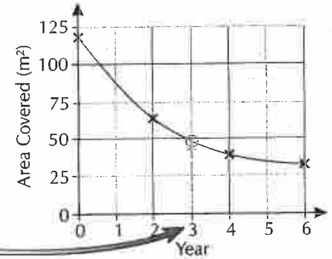


If you want to find the initial rate of reaction, you need to position your ruler at the point where the time is O and estimate where the curve would continue if it carried on below zero.

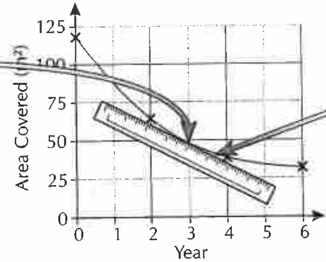
Rates and Tangents

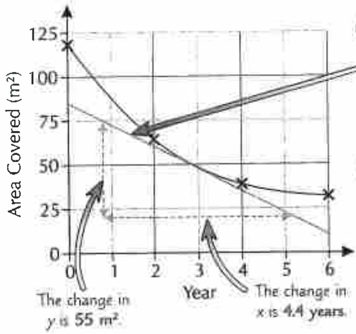
Worked Example

The graph on the right shows the area covered by native British bluebells in a section of woodland, over a period of six years.
What was the rate of population decline in year 3?



1. ***Find the point on the curve that you need to look at.***
   The question asks about the rate of decline in year 3, so find 3 on the x-axis and go up to the curve from there.
2. ***Place a ruler at that point so that it's just touching the curve.***
   Position the ruler so that you can see the whole curve.
3. ***Adjust the ruler until the space between the ruler and the curve is equal on both sides of the point.***





4. ***Draw a line along the ruler to make the tangent.***
   Extend the line right across the graph — it'll help to make your gradient calculation easier as you'll have more points to choose from.
5. ***Calculate the gradient of the tangent to find the rate.***
   You can work out the gradient with this formula:

   gradient = change in y ÷ change in x

   55 m² ÷ 4.4 years = 12.5 m² year⁻¹

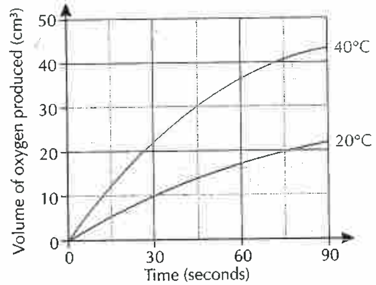
Don't forget the units — you've divided m² by years, so it's m² year⁻¹.

Practice Question

Q1 Carol was investigating the effect of temperature on the rate that catalase breaks down hydrogen peroxide. She used the volume of oxygen produced as a measure of the progress of the reaction. The graph on the right shows her results.

a) What was the rate of oxygen formation at 45 seconds for the reaction at 40 °C?

b) What was the rate of oxygen formation at 30 seconds for the reaction at 20 °C? Give your answer in cm³min⁻¹ to 1 decimal place.



My science teacher was always going off on a tangent...

Drawing a tangent may look tricky, but all it takes is a ruler and a bit of practice. Then you can treat it just like a normal gradient calculation. Just remember that the tangent can only give you the rate for the point on the curve that it's touching.

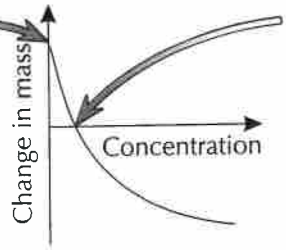
Intercepts

*A graph may have a y **intercept**, an x **intercept** or both. They're really easy to find if you know where to look...*

An **Intercept** Is Where the **Line Crosses** the **Axis**

The *y* **intercept** is where the line crosses the *y*-axis.

There's more about how you can use the *y* intercept on the next page.



The *x* **intercept** is where the line crosses the *x*-axis.

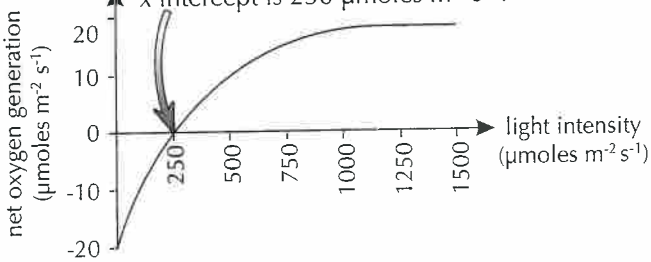
When he set fire to his neighbour's rhododendrons, Jeff knew he had crossed a line.

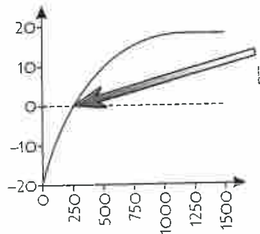
The **x Intercept** is Where **y = 0**

If you need to find the ***x* intercept** of a graph, just do this:

1) Check that the x-axis is positioned where $y = 0$.
2) Find the point where the line **crosses the *x*-axis.**

For example, on this graph the *x* intercept is 250 µmoles m^{-2} s^{-1}.

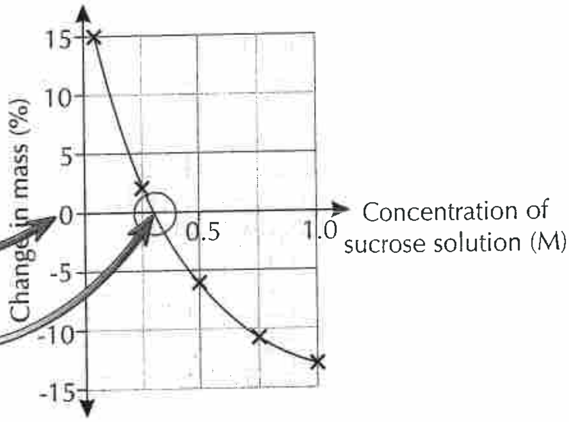




The graph on the left was drawn with the x-axis at the bottom, but you need to read the x intercept as the point on the line where $y = 0$, as shown by the dotted line.

Worked Example

The graph on the right shows a calibration curve to find the water potential of potato cells. The water potential of the cells is equal to the value of the *x* intercept.
What is the water potential of the potato cells?

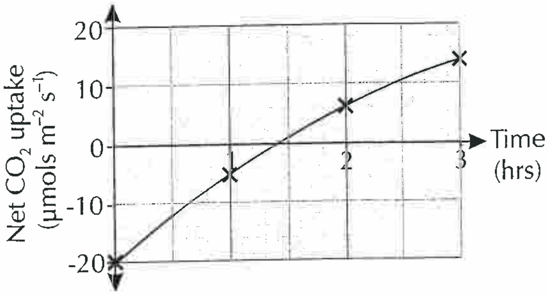


1. ***Look along the x-axis (where y = 0).***
2. ***Read the value on the x-axis where the line crosses it.***

The water potential of the potato cells is **0.3 M**.

Practice Question

Q1 The graph on the right shows the net CO_2 uptake of a plant over the course of three hours. The compensation point is found where the line crosses the x-axis. Give the time when the plant reached its compensation point.



Avoid x when you can — it's such a cross letter...

Don't worry, I haven't forgotten about the y intercept. Just have a little look at the next page... go on...